J. Fluid Mech. (1979), *vol.* 91, *part* 3, *pp.* 493–495
Printed in Great Britain

Separation from spherical caps in Stokes flow

By R. COLLINS

Department of Mechanical Engineering, University College London

(Received 17 July 1978)

Flow-visualization experiments for Stokes flow past thin spherical caps show separation from the rims of the caps in agreement with Dorrepaal, O'Neill & Ranger's (1976) theoretical prediction.

1. Introduction

Solutions to problems involving Stokes flow around bodies of various shapes seem to have been used in the past primarily to determine the forces experienced by the bodies. It is only comparatively recently that they have been studied from the standpoint of the flow patterns desribed and, as a result, several solutions have revealed that separation in Stokes flow is a rather widespread phenomenon. Dorrepaal, Majumdar, O'Neill & Ranger's (1976) axisymmetric solution for a torus, for example, shows separated flow patterns with a system of closed vortices near the centre and a similar feature occurs in the axisymmetric flow past two spheres investigated by Davis, O'Neill, Dorrepaal & Ranger (1976). Further, Dorrepaal, O'Neill & Ranger's (1976) solution for a thin spherical cap reveals separation from the rim of the cap with a vortex ring attached to the concave surface and Michael & O'Neill (1977) have deepened this study by considering the situation when the body is a spherical lens. The phenomenon is not restricted to axisymmetric flows, for Dorrepaal (1978) has shown separation to exist on certain indented cylinders in plane flow. No experimental verification of the existence of separation in a Stokes flow appears to have been published hitherto. In this paper, flow-visualization experiments on thin spherical caps with various cap angles are shown to confirm the flow patterns predicted by Dorrepaal, O'Neill & Ranger's (1976) analysis and reveal separation from the cap rims.

2. Apparatus

A tall tank whose cross-section was a square of side 45 cm was filled with glycerol to a depth of approximately 2·2 m. The caps were initially placed at a position about 10 cm below the free surface gripped in a device of trifoliate form, the leaves of which moved radially and symmetrically on opening. They were released so as to fall on or near the centre-line of the tank. Observations of the flow patterns were made through the braced Perspex walls of the tank and photographs were usually taken at a distance of approximately 1·5 m below the free surface.

Accurately formed spherical shells of plastic of diameter 37·6 mm and shell thickness 0·38 mm (high grade table-tennis balls in fact) were used to produce the spherical caps. Since the ball material has a density only slightly greater than that of glycerol,

0022-1120/79/4200-5850 $02.00

spherical caps cut from them will sink very slowly in that liquid. The cap velocity U measured by a stop-watch over a known distance took values from approximately 1·7 mm/s to 7·8 mm/s, so that the Reynolds number $Re = 2Ua/\nu$, where $2a$ is the spherical diameter and ν the kinematic viscosity, ranged from 0·05 to 0·25. The flows observed were thus unequivocally in the Stokes regime.

Visualization of the flow patterns was achieved by exploiting a naturally occurring phenomenon. Glycerol is hygroscopic and water vapour from the atmosphere is thus adsorbed at its free surface. In being brought to its initial position in the tank, a cap penetrated the free surface of the liquid and its own surface thus became coated with with a thin layer of the water-laden glycerol. During the subsequent motion this layer was progressively stripped from the surface and fed into the flow to reveal patterns of streaklines at and very near to the body surface. The liquid forming the streaklines thus had a refractive index different from that of the surrounding liquid, and as a result, these patterns were visible from some observation angles under naturally occurring lighting conditions; with a shadowgraph they were revealed in high contrast. They could be readily seen at the bottom of the tank and the thin vertical column left in the tank delineating the path of the body persisted for several hours. After only a few experiments, the field of view thus became cluttered with tracks from previous experiments and the tank had to be left to clear overnight. Since in addition each transit of the tank could take up to 20 min and small imperfections in manufacture made it difficult to achieve a steady axisymmetric flow with every attempt, obtaining acceptable photographs for various cap angles was time consuming. For a steady flow, streamlines and streaklines coincide, so that this method of visualization may then be expected to show the form of any separation streamline springing from the body.

3. Results

Figure 1 (*a*) (plate 1) shows a control experiment conducted with a complete sphere which had been weighted internally with lead shot and then re-sealed. There is of course no separation in this case. The lateral dimension of the column observed some distance behind the sphere is approximately $\frac{1}{40}\,a$, where a is the sphere radius, and we may expect the edge of this track to correspond to a dimensionless Stokes's stream function $\psi^* = \psi/Ua^2 \simeq 3\times10^{-4}$. The theoretical forms for $\psi^* = 0$ and 3×10^{-4} on the downstream side of the sphere are shown in figure 1 (*b*) (plate 1) and agree well with the observed pattern. The same values of ψ^* have been plotted for comparative purposes for the spherical caps of various angles shown in figures 2–4 (plates 1–3) and agreement between theory and experiment is again generally good, despite the slight asymmetry detectable in all patterns observed. There is no doubt that the flow separates from the rim.

Before embarking on these experiments it was hoped that a quantitative comparison with the theoretical predictions might be made by measuring the streamwise extent of the eddy and the angle of separation from the rim. During the work, however, it became clear that this hope would not be realized because, as both the experimental and theoretical parts of figures 2–4 show, even when the thickness of the tracer material on the cap has been reduced to approximately $\frac{1}{100}a$ this is still too thick to allow the position of the rear stagnation point to be identified with precision. Other attempts to define the separation streamline by making the internal motion in the eddy visible with dyed glycerol were unsuccessful because the introduction of the tracer material by hypodermic syringe at points near the cap rim seemed invariably to lead to unsteady asymmetric motions of the caps. When introduced only at the centre of the concave surface, coloured tracer material did indicate an outward radial motion consistent with separation from the rim, but since the internal motions were so slow the tracer moved only very short distances during the transit of the cap and did not reach the rim. The difficulty in measuring separation angle arises because theoretical patterns giving detail near the rim show localized changes in curvature of the separation streamline there and, since it is the streamline $\psi^* = 3 \times 10^{-4}$ which is being observed, the photographic image has a rounded-off form there. Measurement of separation angles from the photographs is thus susceptible to large error and this detail has not been assessed. The qualitative comparison employed here shows satisfactory agreement between theory and experiment.

The translational velocities of the caps soon reached steady values and the main experimental difficulty arose from an unsteadiness in the attitude of the cap. Sometimes a cap would adopt a large asymmetry (a 10° tilt for example) soon after its release; at other times a very slow oscillation of small amplitude with a period of minutes would be observed. Invariably the tilt and oscillation were much reduced by the time the photographing position was reached but the result in both cases was that the motions were no longer strictly steady, so that the streakline patterns observed could not then be interpreted as streamlines. Some caps had their attitudes improved by gluing small disks of thin lead sheeting at the centre of the concave surface and shaving these away where necessary. From the difficulties encountered in doing this it was concluded that it was not feasible to weight the rim so as to make the cap fall with its concave face downwards in an axisymmetric manner, an experiment which, since Stokes flow is reversible, could be expected to demonstrate the existence of a forward wake. An alternative approach to that experiment might be made if caps formed from a buoyant material could be obtained and allowed to rise in the glycerol. Experience with the present experiments, however, suggests that to obtain the forward wake it would probably be better to constrain the motion of the body and develop an apparatus in which the cap is propelled in the correct attitude while supported by a fine axial rod.

I am indebted to Dr J. M. Dorrepaal for providing the additional data enabling the streamline patterns of figure 2 to be plotted and to the National Research Council of Canada for providing financial resources for the apparatus.

REFERENCES

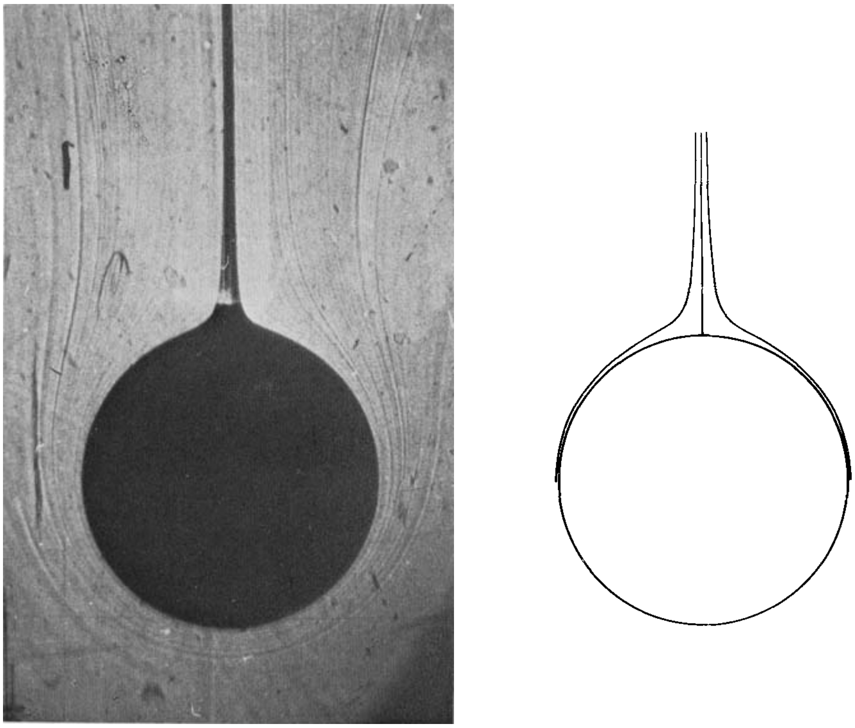
Davis, A. M. J., O'Neill, M. E., Dorrepaal, J. M. & Ranger, K. B. 1976 Separation from the surface of two equal spheres in Stokes flow. *J. Fluid Mech.* **77**, 625–644.

Dorrepaal, J. M. 1978 Stokes flow past a smooth cylinder. *J. Engng Math.* **12**, 177–185.

Dorrepaal, J. M., Majumdar, S. R., O'Neill, M. E. & Ranger, K. B. 1976 A closed torus in Stokes flow. *Quart. J. Mech. Appl. Math.* **29**, 381–397.

Dorrepaal, J. M., O'Neill, M. E. & Ranger, K. B. 1976 Axisymmetric Stokes flow past a spherical cap. *J. Fluid Mech.* **75**, 273–286.

Michael, D. H. & O'Neill, M. E. 1977 The separation of Stokes flows. *J. Fluid Mech.* **80**, 785–794.



(*a*) (*b*)

FIGURE 1. Flow patterns for a sphere in Stokes flow.
(*a*) Experimental. (*b*) Theoretical.

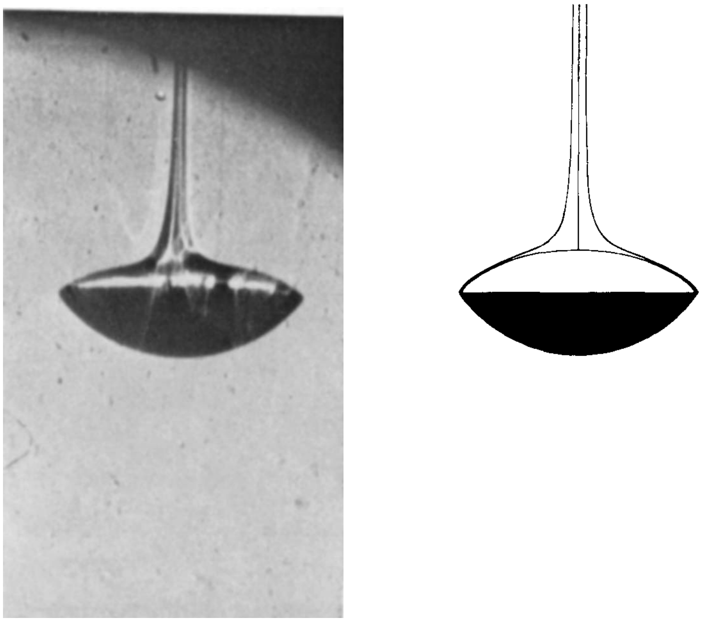
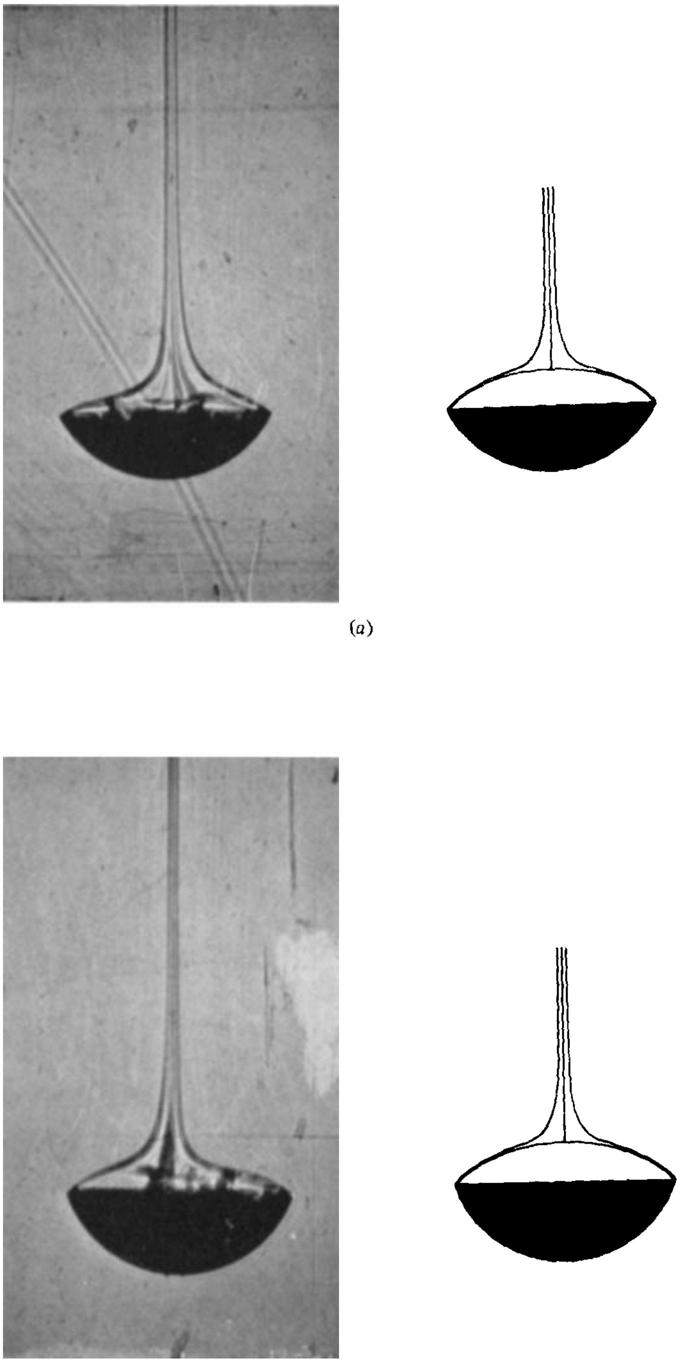


FIGURE 2. Flow patterns for a spherical cap in Stokes flow; $\alpha = 54{\cdot}6^\circ$, where α is the semi-angle subtended by the cap at the centre of the sphere.



(*a*)

(*b*)

FIGURE 3. Flow patterns for spherical caps in Stokes flow. (*a*) $\alpha = 62{\cdot}2°$. (*b*) $\alpha = 70{\cdot}6°$.

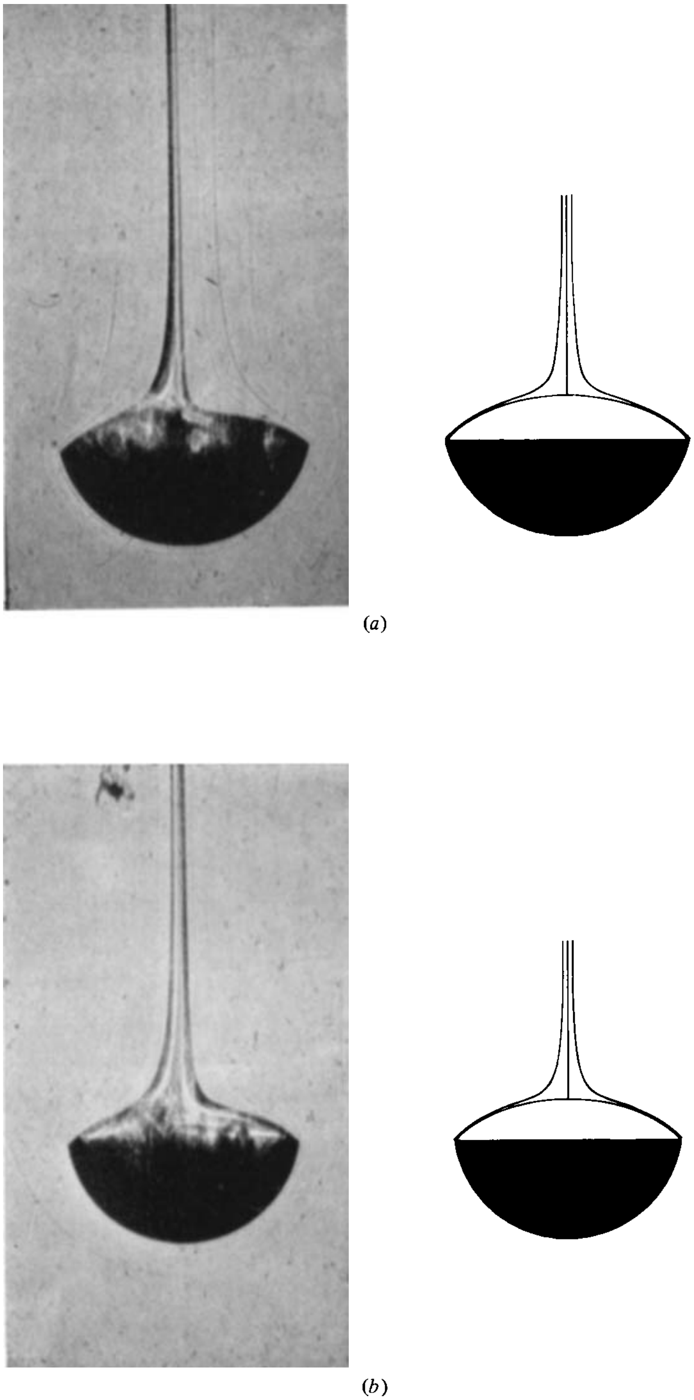


FIGURE 4. Flow patterns for spherical caps in Stokes flow. (*a*) $\alpha = 75.8°$. (*b*) $\alpha = 82{\cdot}4°$.